A MATHEMATICAL INTERPRETATION OF *I CHING* (易経)

Maiko Yamamori
MA, Kyoto University

Preface

- Are eastern thoughts less rational than western?
- Are they irrational?

Preface

- What does ‘rational’ mean?
  - As you know, the word ‘rational’ has its origin in **ratio**.
  - Numbers or mathematical notions has been models of rationality.

- Then, aren’t there such thoughts in East?
- →There are! Especially *I Ching* (易経).

Preface

- The aim of this presentation;
  - To oppose the idea that eastern thoughts are irrational.
  - To see that *I Ching* can be interpreted that it commits to mathematical notions no less closely than many western thoughts.

  - Subaim:
  - To show that in East there is other thought that can be interpreted mathematically but Buddhism.

Outline of this presentation

- 1: Commitment to mathematical notions in West
- 2: Ideas of *I Ching*
- 3: From the mathematical point of view

Outline of this presentation

- 1: Commitment to mathematical notions in West
- 2: Ideas of *I Ching*
- 3: From the mathematical point of view

1: Commitment to mathematical notions in West

- We know that many western thoughts relate to mathematical notions, for example numbers or polyhedrons.

- We can classify into two cases how a thought relates to mathematical notions.
  1. Symbolization: mathematical notions used in a thought symbolize something in the world.
  2. Structure: a thought has mathematical structure, for example order structure.

- We'll see examples.

1: Commitment to mathematical notions in West

1. Symbolization
2. Structure
3. Conclusion

- In some western thoughts, mathematical notions are used to symbolize something.
- Let's see some examples.

1: Commitment to mathematical notions in West

1. Symbolization
2. Structure
3. Conclusion

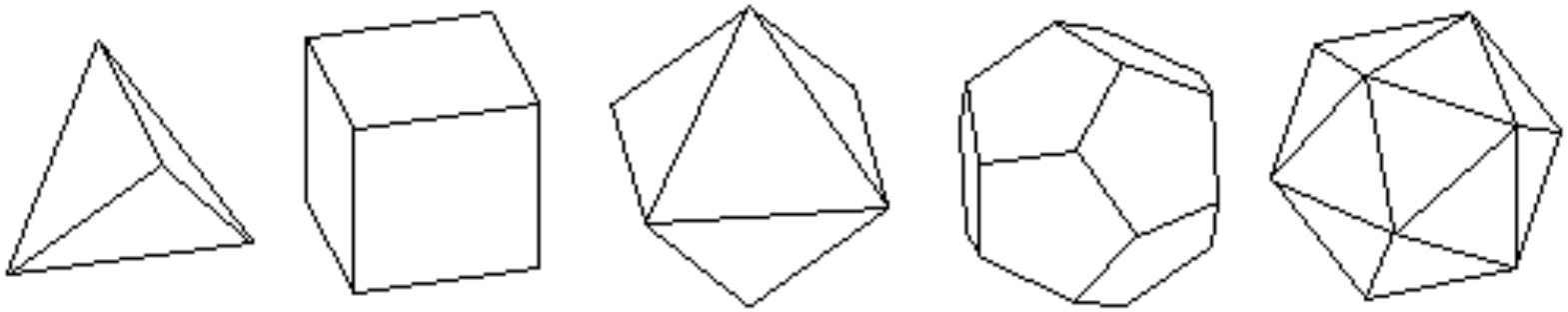
- Pythagoras (BC571-495)
  - Pythagoras thinks that numbers symbolizes anything, because the universe is made by numbers.
    - 1 means absoluteness
    - 2 means relativeness
    - 3 means development
    - and so on.
    - 10 is the complete number and symbolize completeness.

1: Commitment to mathematical notions in West

1. Symbolization
2. Structure
3. Conclusion

- Plato (BC427- 347)
  - Plato connects regular polyhedrons to four-elements.
    - According to Plato, there are four elements; fire, earth, air and water.
    - And there are five regular polyhedrons in the world. (This is a proved mathematical fact.)
  - He thinks that each of polyhedrons symbolizes one element.

1: Commitment to mathematical notions in West



Tetrahedron: Fire

Hexahedron: Earth

Octahedron: Air

Dodecahedron: (Universe)

Icosahedron: Water

1: Commitment to mathematical notions in West

1. Symbolization
2. Structure
3. Conclusion

- Kepler (1571-1630)
  - This astronomer thinks that each of regular polyhedrons symbolizes extraterrestrial planets.
    - Octahedron: Mercury
    - Icosahedron: Venus
    - Dodecahedron: Mars
    - Tetrahedron: Jupiter
    - Hexahedron: Saturn

1: Commitment to mathematical notions in West

1. Symbolization
2. Structure
3. Conclusion

- Now we'll see structures of some western thoughts.
- Bourbaki introduced three notions of structures into mathematics; algebraic structure, order structure and topological structure.
  - Intuitive explanations
  - Order structure : Orders defined on a set
  - Algebraic structure: Operations defined on a set
- Some western thoughts have algebraic structure.

1: Commitment to mathematical notions in West

1. Symbolization
2. Structure
3. Conclusion

- Plato (BC427- 347)
  - Plato thinks we can continue to classify everything by two groups.
  - This has an algebraic structure; mereological structure.
    - ※ mereology is the theory of relations between part and whole, part and part.

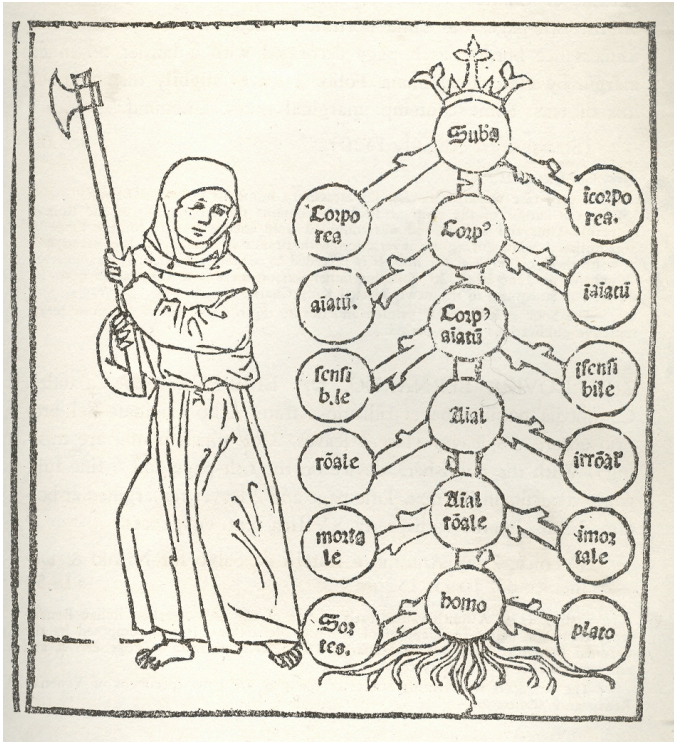
1: Commitment to mathematical notions in West

--- non-being
--- being
        |
        --- non-living
        --- living
                |
                --- aquatic
                --- land
                        |
                        ---many-legged
                        ---two-legged (human)

1: Commitment to mathematical notions in West

1. Symbolization
2. Structure
3. Conclusion

- Porphyrian tree
  - This is a diagram to illustrate how to divide notions.
  - This also can be interpreted as mereological structure.



1: Commitment to mathematical notions in West

1. Symbolization
2. Structure
3. Conclusion

- In West, people had refined these notions and they get today's frameworks of thoughts, for example logic.
- It is not too much to say that they are prototype of western rational thought.

- (When you say that eastern thoughts are irrational you mean that eastern thought don't commit to such notions, it is wrong… We'll see it later.)

1: Commitment to mathematical notions in West

1. Symbolization
2. Structure
3. Conclusion

- The relation between thoughts and mathematical notions can be classified into two cases.
  1. Mathematical notions symbolize something.
  2. A thought has rational structure.

1: Commitment to mathematical notions in West

1. Symbolization
2. Structure
3. Conclusion

1. Mathematical notions symbolize something.
2. A thought has rational structure.

- *I Ching* satisfies both!

Outline of this presentation

- 1: Commitment to mathematical notions in West
- 2: Ideas of *I Ching*
- 3: From the mathematical point of view

2. Ideas of *I Ching*

- I make brief introduction of the thought of *I Ching*.

- *I Ching* has some organizations. And symbols in the organizations express something.
- We can see that *I Ching* satisfies the condition 1.
  - “Mathematical notions (numbers) symbolize something.”

2: Ideas of *I Ching*

1. What does *'I Ching' means*?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- The original name of the book which is called *I Ching* is *I* (易).
- This book was used as the most important scripture of Confucianism, and added the term *Ching* (経) that means scripture.
- *I Ching* had has the authority in Chine like the Bible in Western for two thousands years.

2: Ideas of *I Ching*

1. What does *'I Ching' means*?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- What is the meaning of the word *I* (易)?
- According to Zheng Xuan (鄭玄: 127-200), *I* has three meanings.
    1. Easiness; plainness (簡易)
    2. Changeability (変易)
    3. Unchangeability (不易)

    - Everything (both nature and human affairs) is changeable. There is, however, unchangeable regularity in this ceaseless transition. Because of this regularity, it is easy and plain to understand and follow the law of Nature.

2: Ideas of *I Ching*

1. What does *'I Ching' means*?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- *I Ching* is…

1. a mysterious text for divination
    - You may think "*I Ching* seems to be too mysterious and not rational, because it is a divination's text!". Please wait.
2. a philosophical text which tells us the principle or regularity both of universe and human life.

2: Ideas of *I Ching*

1. What does *'I Ching' means*?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- What is the philosophy in *I Ching*?
- Let's see the fundamental notions in it.

2: Ideas of *I Ching*

1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- *Taiji* (太極)
    - Taiji is a chaos that is the source of everything in the universe.
        - This is complete chaos and it is so difficult to explain or express it.

2: Ideas of *I Ching*

1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- *Liangyi* (両儀)
    - From *Taiji*, two natural enagies *qi* (気) arise;
    - *yin* (陰, - -) and *yang* (陽, 一).
    - A pair of *yin* and *yang* is called *Liangyi.*
    - The Dualism of *yin* and *yang* is the base notion of *I Ching*.

2: Ideas of *I Ching*

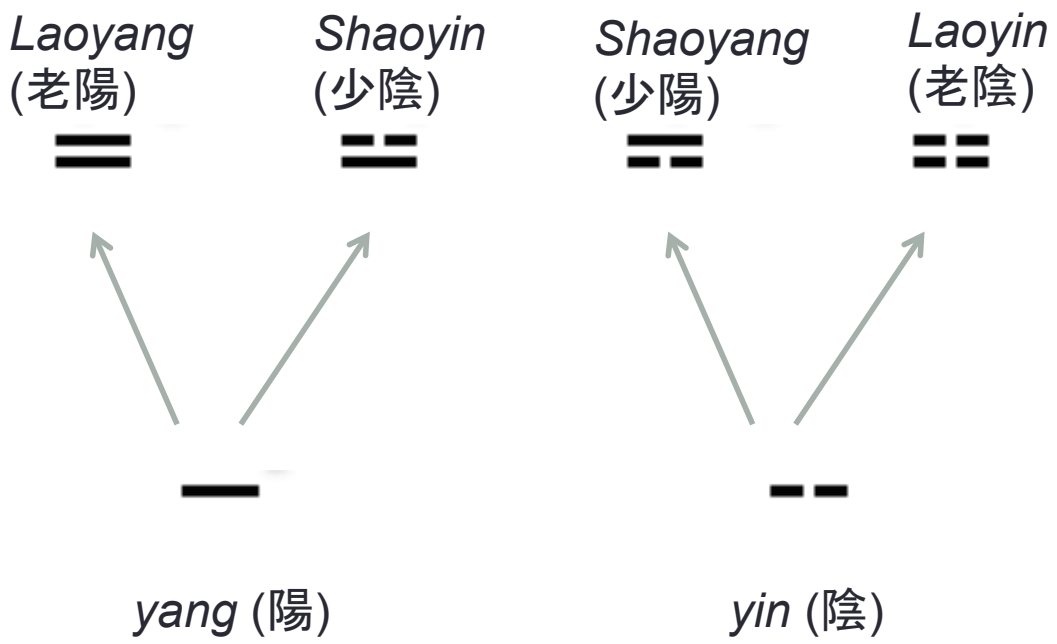
1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- Dualism of *yin* and *yang*;
    - every phenomenon in the universe occurs by the formation and transition of *yin* and *yang*.
    - Any interpretation of them is not unanimous; but it is clear that they symbolize opposing notions but they don't hostile each other.
        - *yin* symbolizes female, ruled people, passiveness, even numbers and so on.
        - *yang* symbolizes male, rulers, aggressiveness, odd numbers and so on.

2: Ideas of *I Ching*

1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
   1. *Taiji*
   2. *Liangyi*
   3. *Sixiang*
   4. *Bagua*
   5. Hexagram
   6. Some oppositions
4. Conclusion

- *Liangyi* generates *Sixiang*(四象).
  - *Xians* are made by putting *yin* or *yang* line on the top of *yin* or *yang* lines.



2: Ideas of *I Ching*

1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
   1. *Taiji*
   2. *Liangyi*
   3. *Sixiang*
   4. *Bagua*
   5. Hexagram
   6. Some oppositions
4. Conclusion

- *Sixiang* generates *Bagua*(八卦).
  - *gua*s are made by putting *yin* or *yang* line on the top of each *xiang*s.

乾 兌 離 震 巽 坎 艮 坤

Qian Dui Li Zhen Xun Kan Gen Kun

Laoyang *Shaoyin* *Shaoyang* *Laoyin*

老陽 少陰 少陽 老陰

2: Ideas of *I Ching*

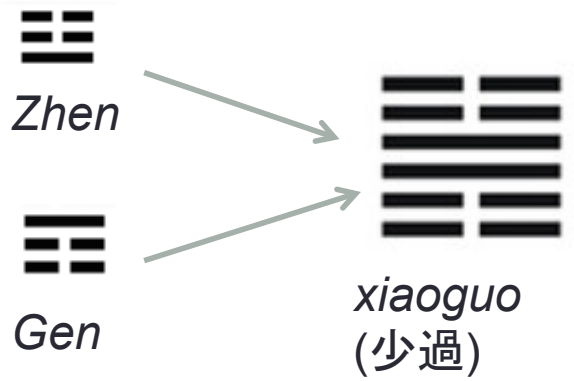
1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- *Bagua* was made to represent everything in the universe.
    - And each *gua* symbolize many things;
        - Ex. *Qian* (乾) symbolizes
            - Heaven
            - Father
            - Soundness
            - Horse
            - Neck
            - Northwest …

2: Ideas of *I Ching*

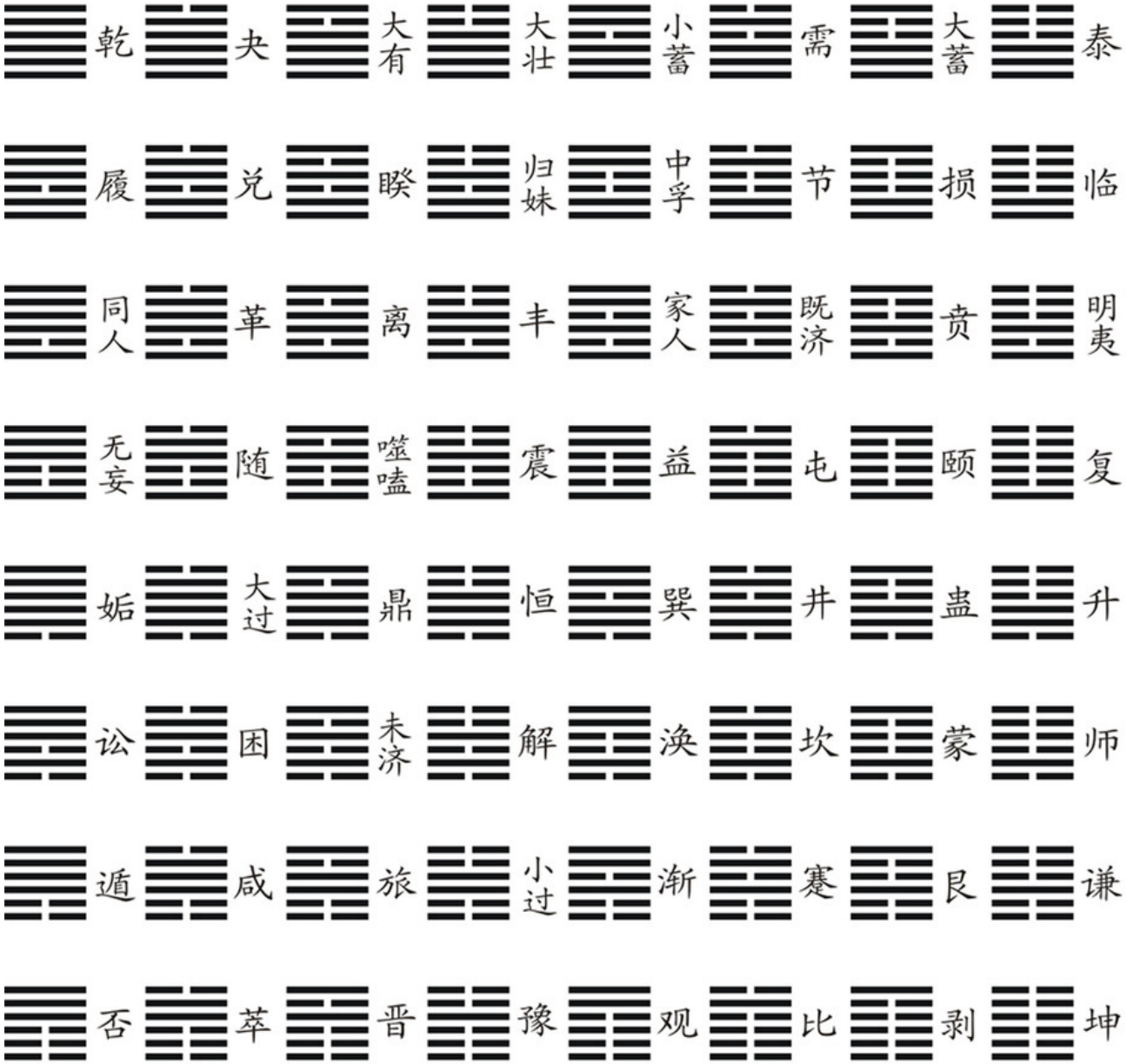
1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- *Bagua* is not enough to describe subtle phenomena.
- So put two *gua*s one on top of the other, and hexagrams (六十四卦) appear.
    - Ex. By putting *Zhen* on top of *Gen*, we get *xiaoguo* (少過).



2: Ideas of *I Ching*

1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
   1. *Taiji*
   2. *Liangyi*
   3. *Sixiang*
   4. *Bagua*
   5. Hexagram
   6. Some oppositions
4. Conclusion



2: Ideas of *I Ching*

1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- Note:
    - In *I Ching,* the word 'hexagram' is the name of a figure consists of six lines.
        - It is not the name of  .

- Each hexagrams symbolizes what is one's fate, what should one do, what is the best direction, and so on.
- And by hexagrams, everything in the universe is expressed in detail.

2: Ideas of *I Ching*

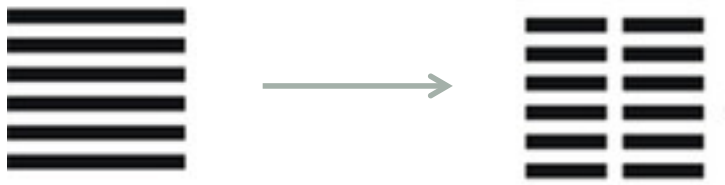
1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- There are not only one notion but many notions of oppositions in *I Ching*.
- We'll see the most important and famous two oppositions.
    1. *Pang-tong gua*
    2. *Fan-dui gua*

2: Ideas of *I Ching*

1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

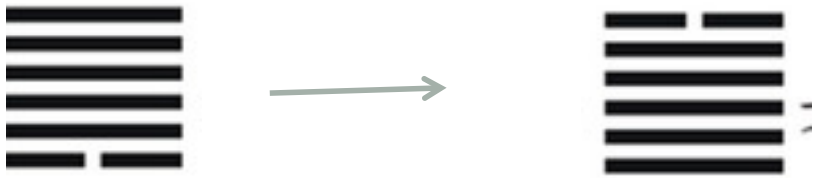
- 1. *pang-tong gua* (旁通卦)
    - This is the result of exchanging all *yin* line with *yang* line, and *yang* line with *yin* line in one hexagrams.
    - This shows what lies behind things.



2: Ideas of *I Ching*

1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- 2. *fan-dui gua* (反対卦)
    - This is the result of turning given *hexagram* upside down.
    - This shows hexagrams saw from the opposite side.
    - so this expresses how the other person see things.



2: Ideas of *I Ching*

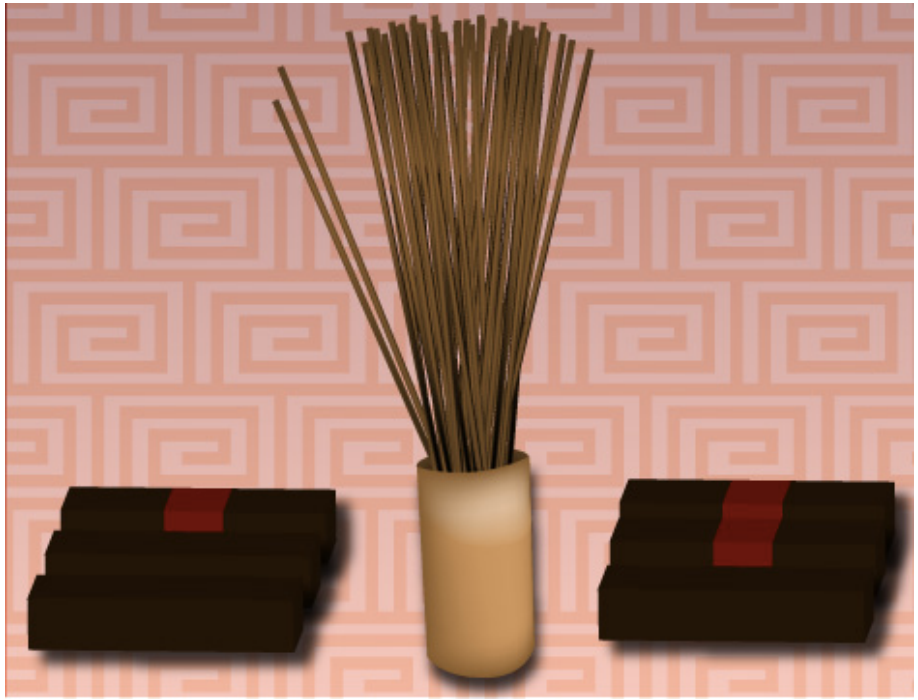
1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- sum:
- From *Taiji* (太極), *Liangyi* (両儀) arises.
- *Liangyi* generates *Sixiang* (四象).
- *Sixiang* generates *Bagua* (八卦).
- By overlapping two *gua*s, we can get hexagrams (六十四卦). Hexagrams represent everything in the world.

2: Ideas of *I Ching*

1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- As above, *I Ching* is a text for divination, too.
- We use hexagrams in the divination. The divination gives one hexagram which symbols your fate.
- The way of this divination is, in a word, to get numbers. Numbers represent symbols.



2: Ideas of *I Ching*

1. What does *'I Ching'* means?
2. The characteristics
3. The ideas of *I Ching*
    1. *Taiji*
    2. *Liangyi*
    3. *Sixiang*
    4. *Bagua*
    5. Hexagram
    6. Some oppositions
4. Conclusion

- Sum of sum:
- Hexagrams symbolize everything in the world.
- Numbers represent hexagrams.
- So we can say that numbers (at least indirectly) symbolize everything in the world.

- Next question:
    - Then, do these diagrams have rational structures? (otherwise we don't want to say *I Ching* is rational from the point of mathematical view.)
    - →Yes

Outline of this presentation

- 1: Commitment to mathematical notions in West
- 2: Ideas of *I Ching*
- 3: From the mathematical point of view

3: From the mathematical point of view

- As above, *Liangyi* generates *Sixiang*, *Sixiang* generates *Bagua* and *Bagua* generates hexagrams.

- As Leibniz noticed, the ways to generate them are so systematic.
- Therefore you can interpret them as lattices.
- We can find that *I Ching* satisfies the second condition;
  - “A thought has rational structure.”

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
    1. *Liangyi*
    2. *Sixiang*
    3. *Bagua*
    4. *Hexagram*
3. Oppositions
    1. *Pang-tong gua*
    2. *Fan-dui gua*
4. Conclusion

- What is lattice?
    - The reason why I (and others) want to interpret organizations in *I Ching* as lattice is that a lattice has the important role in mathematics.
    - Definition: A lattice is a non-empty poset in which any two elements have a unique supremum and a unique infimum.
    - Anyway, a lattice is a special set.

3: From the mathematical point of view

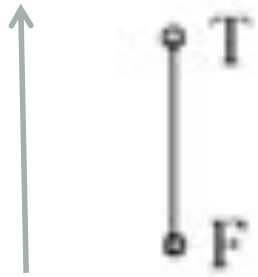
1. What is lattice?
2. They are lattices
    1. *Liangyi*
    2. *Sixiang*
    3. *Bagua*
    4. *Hexagram*
3. Oppositions
    1. *Pang-tong gua*
    2. *Fan-dui gua*
4. Conclusion

- Bourbaki takes three structures as fundamental structure in terms of mathematics.
    1. Algebraic structure
    2. Order structure
    3. Topological structure
- A lattice has algebraic structure and order structure.
- There are lattices around you, especially logic.

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
    1. *Liangyi*
    2. *Sixiang*
    3. *Bagua*
    4. *Hexagram*
3. Oppositions
    1. *Pang-tong gua*
    2. *Fan-dui gua*
4. Conclusion

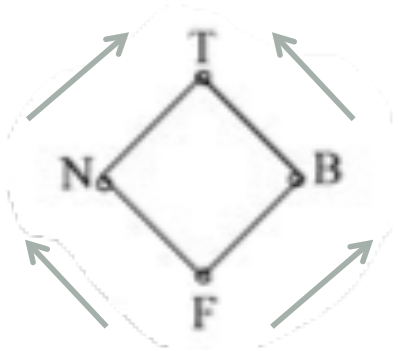
- Classical logic:
    - This is very trivial example;
    - The truth values of classical logic form (boolean) **Lattice**.



3: From the mathematical point of view

1. What is lattice?
2. They are lattices
    1. *Liangyi*
    2. *Sixiang*
    3. *Bagua*
    4. *Hexagram*
3. Oppositions
    1. *Pang-tong gua*
    2. *Fan-dui gua*
4. Conclusion

- Non-classical logic:
    - For example, the truth values of FDE (first-degree entailment) form (de Morgan) **Lattice**.
    - As you know, this is used in interpreting Buddhist logic.



3: From the mathematical point of view

1. What is lattice?
2. They are lattices
    1. *Liangyi*
    2. *Sixiang*
    3. *Bagua*
    4. *Hexagram*
3. Oppositions
    1. *Pang-tong gua*
    2. *Fan-dui gua*
4. Conclusion

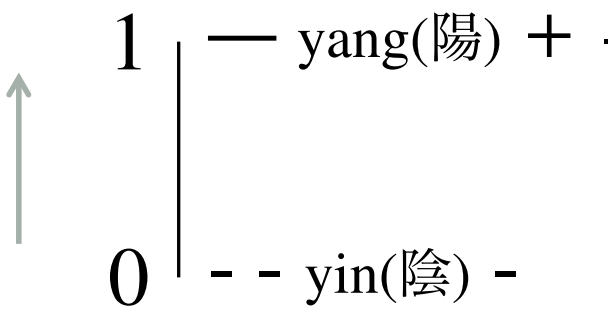
- *Liangyi, Sixiang, Bagua* and hexagrams can be interpreted as lattices.
    - Zhang Qingyu (1994) tries to show it, but by his method he can't get the structure he wants to get.
    - So we use another method.

- Zhang Qingyu (张清宇) (1994) '易图的内涵格解释' "中国研究" 1994

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

- As above, *Liangyi* consists of *yin* (- -) and *yang* (—).
  - Let yin be the lower and yang be the upper.
  - we can use 1 to represent *yang* and 0 to represent *yin*. These are their valuations.

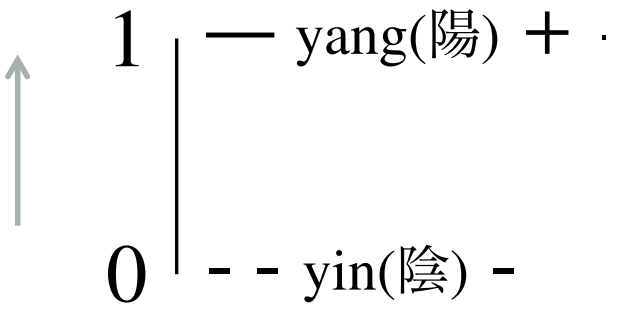


- This is a lattice.

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

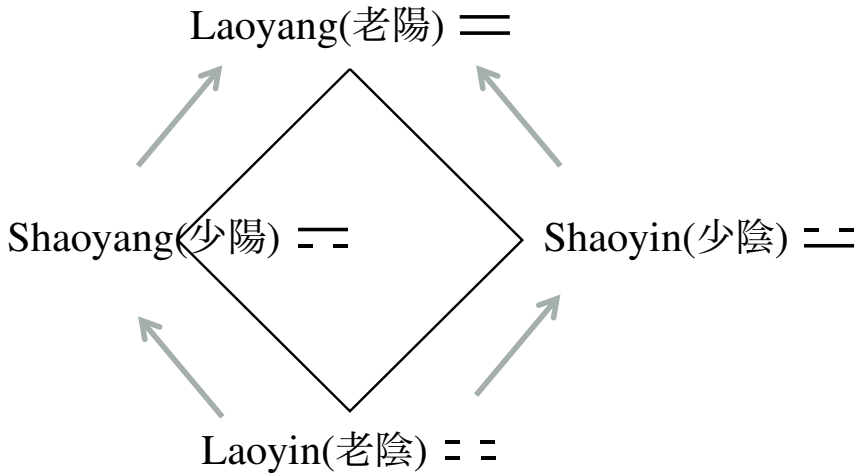
- The reason why *yang* is 1 and *yin* is 0:
  - Essentially, their worth are equal.
  - However it is said that *yin* and *yang* symbolize many things, including small and big.
    - *yin* is small and *yang* is big.
  - Therefore it is natural to interpret that *yang* is bigger than *yin*.



3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

- *Sixiang* consists of ⚋, ⚊, ⚋, ⚊ .
- We can interpret *Sixiang* as follow;
  - One *xiang* α is less or equal to one xiang β, iff, the upper line and lower line of α are less or equal to those of β respectively.
  - (Use the order in *Liangyi* (*yin* < *yang*).)

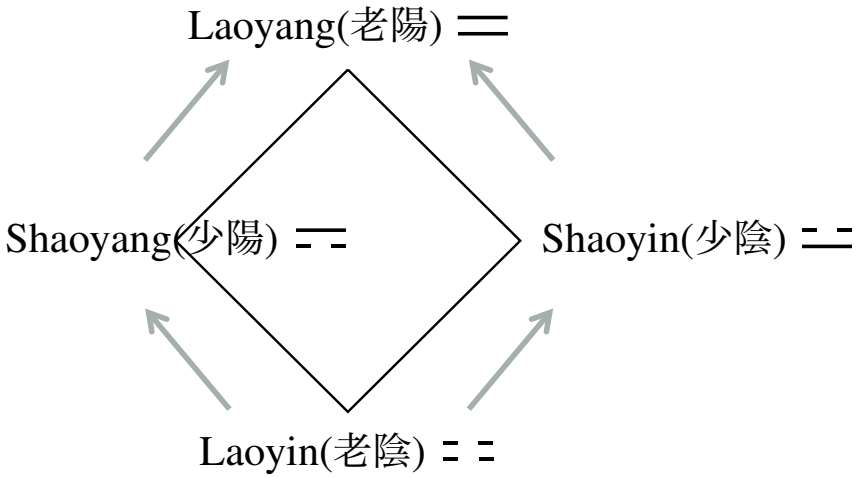


- This is a lattice.

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

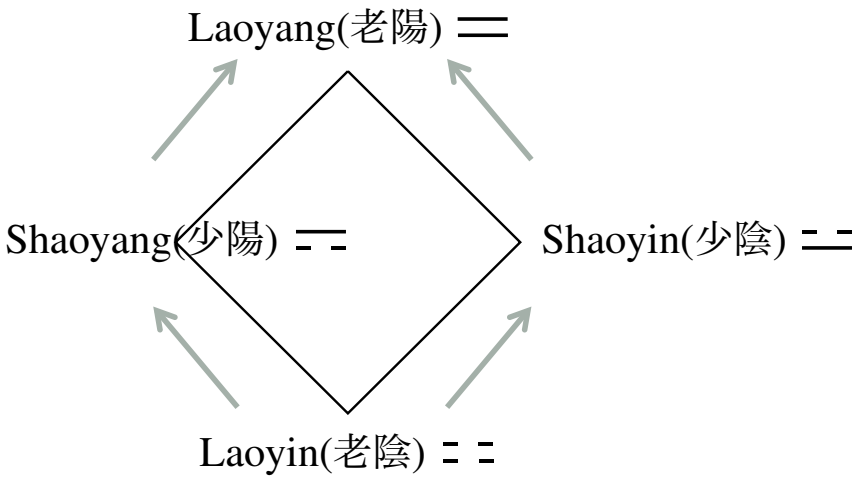
- In fact, there is no justification for ordering *xiang*s now.
- One reason is that at first there is no unanimous interpretation of *Sixiang*.
- However this can be justified when we see *Bagua*'s order.
- Moreover, because *yang* is bigger than *yin* as above, it is natural to order *xiang*s by the number of *yang* lines.
  - So we can't compare *xiang*s which are same in the number of *yang* lines.



3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

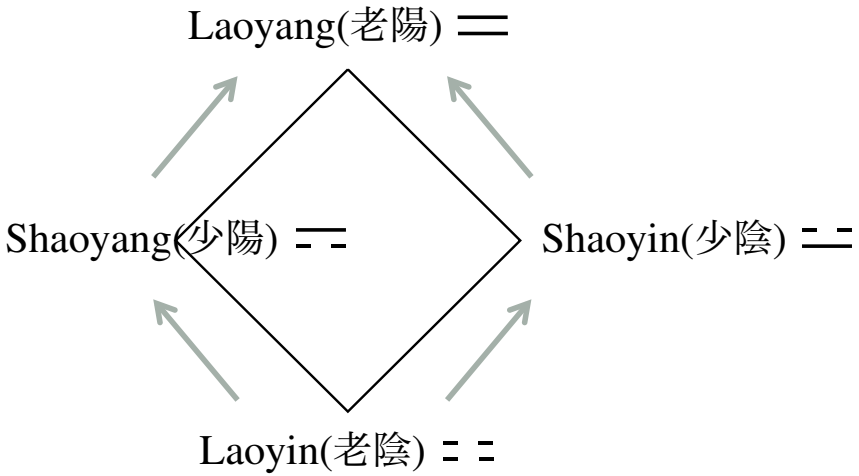
- The interpretaions:
  - Each *xiangs* consists of two lines, so we can express each of them as ordered lists of two elements (2-tuples).
    - *Laoyin* is <*yin, yin*>
    - *Shaoyang* is <*yin, yang*>
    - *Shaoyin* is <*yang, yin*>
    - *Laoyang* is <*yang, yang*>



3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

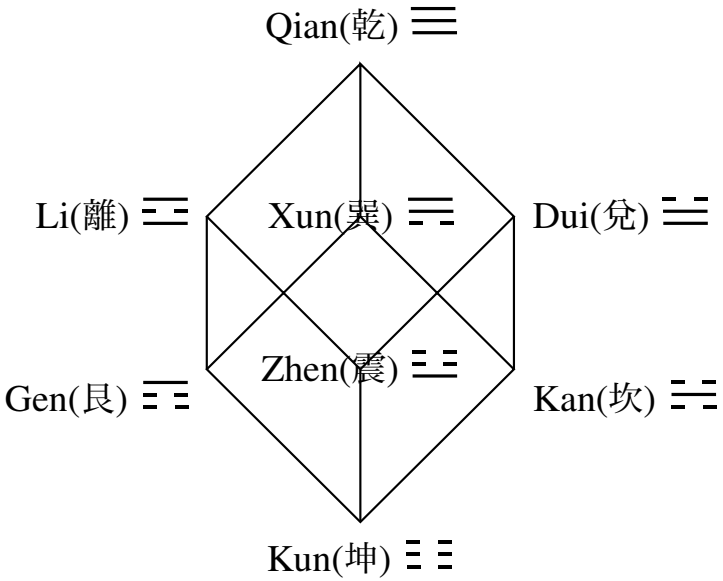
- The interpretaions:
  - Like as in *Liangyi*, we can interpret *yin* as 0 and *yang* as 1. Then,
    - *Laoyin (<yin, yin>)* is <0,0>.
    - *Shaoyang* (*<yin, yang>)* is <0,1>.
    - *Shaoyin* (*<yang, yin>)* is <1,0>.
    - *Laoyang* (*<yang, yang>)* is <1,1>.



3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

- *Bagua* consists of eight *gua*s.
- We can interpret *Bagua* as follow;
  - One *gua* α is less than or equal to a *gua* β, iff, α's upper, middle and lower lines are less than or equal to β's respectively.
  - Use the order in *Liangyi* (*yin* < *yang*).

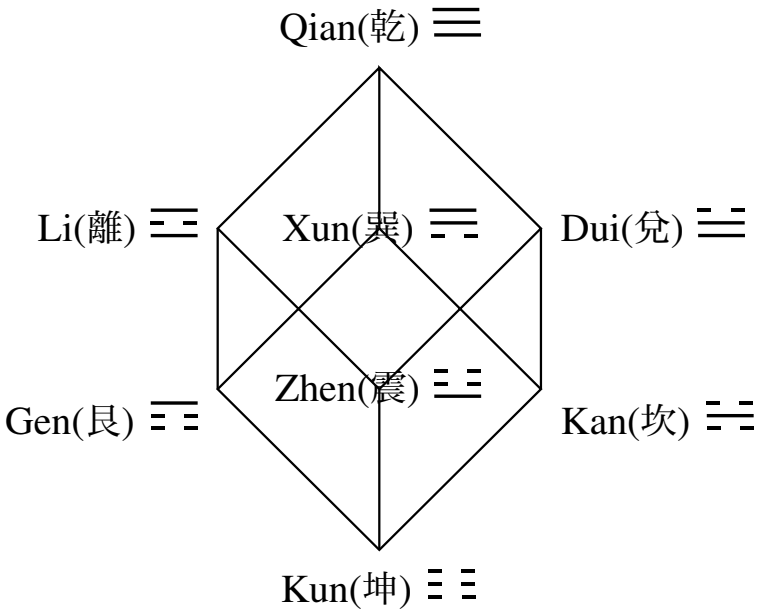


- This is a lattice.

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

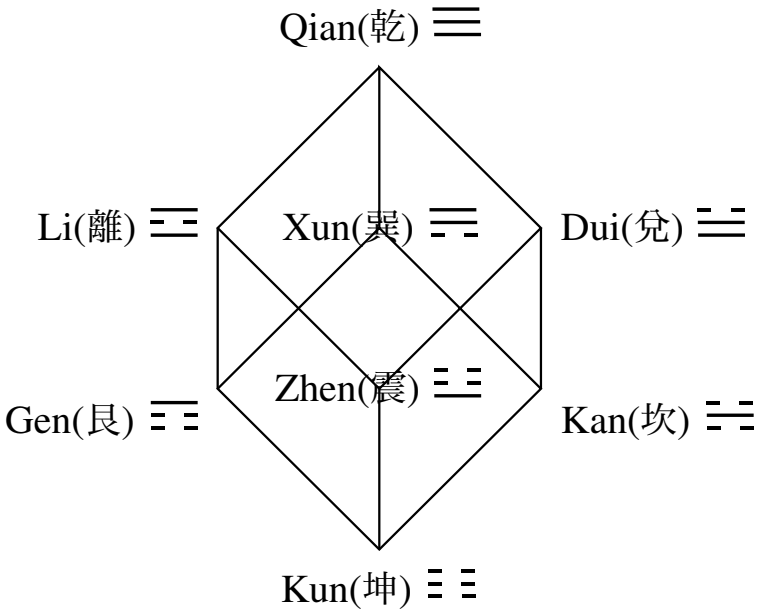
- According to *I Ching*, "*Qian* stand in top and *Kun* in bottom".
- So it is natural to take them as the greatest and the least.
- And as above, *yang* is bigger than *yin*, so we can order *gua*s by the number of *yang* lines.
  - So we can't compare *gua*s which are same in the number of *yang* lines.



3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

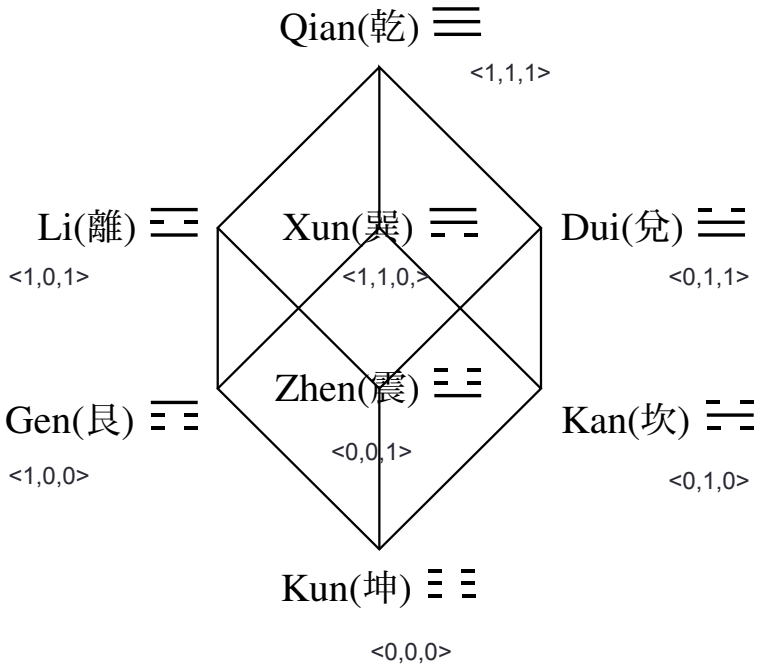
- The interpretations:
  - We use the same way as in *Sixiang*.
  - Each of *gua*s consists of three lines, so we can express them as 3-tuples.
    - Ex. *Dui* is <*yang, yang, yin*>.



3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

- The interpretations:
  - Like as *Liangyi* again, we can interpret *yin* and *yang* as 0 and 1 respectively.
    - Ex. *Dui* (<*yang, yang, yin*>) is <1,1,0>.



3: From the mathematical point of view

1. What is lattice?
2. They are lattices
    1. *Liangyi*
    2. *Sixiang*
    3. *Bagua*
    4. *Hexagram*
3. Oppositions
    1. *Pang-tong gua*
    2. *Fan-dui gua*
4. Conclusion

- According to Zhang Qingyu, there are two ways to generate hexagrams.
    1. Overlapping six lines simply.
    2. Overlapping two *gua*s.

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

- According to Zhang Qingyu, there are two ways to generate hexagrams.
  1. Overlapping six lines simply.
  2. Overlapping two *gua*s.

- 2 is more natural in terms of the story of *I Ching*.
  - *Liangyi* generates *Sixiang, Sixiang* generates *Bagua,* and hexagrams are made by two *gua*s.
- However, in order to continue to use our way as above, 1 is better. So consider only 1.
- (In any case, their Hasse diagrams are the same.)

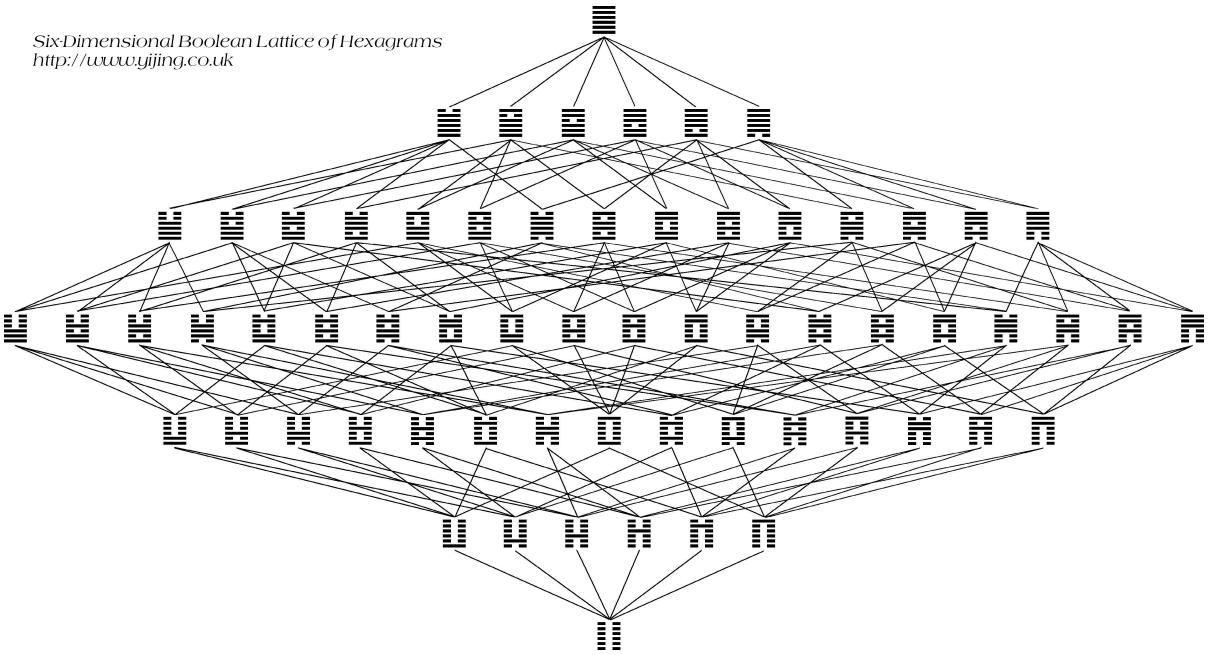
3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

- Overlapping six lines:
  - Each hexagrams consists of six lines. (*yin* (- -) or *yang* (—))
  - So we can interpret as follow;
    - a hexagram α is less than or equal to a hexagram β, iff, α's six lines (bottom line, second line, third line, forth line, fifth line and top line) are less than or equal to β's respectively.
    - Use the order in *Liangyi* (*yin* < *yang*).

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion



3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

- The interpretations:
  - We use the familiar way again;
  - Each hexagrams consists of six lines, so we can interpret them as 6-tuples.
    - Ex. *zhongfu* consists of *yang* as bottom line, *yang* as second line, *yin* as third line, *yin* as forth line, *yang* as fifth line and *yang* as top line.
    - So it can be expressed as <*yang, yang, yin, yin, yang, yang*>.

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

- The interpretations:
  - And as usual, replace *yin* and *yang* with 0 and 1 respectively.
    - Ex. *Zhongfu* (<*yang, yang, yin, yin, yang, yang*>) is <1,1,0,0,1,1>.

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
    1. *Liangyi*
    2. *Sixiang*
    3. *Bagua*
    4. *Hexagram*
3. Oppositions
    1. *Pang-tong gua*
    2. *Fan-dui gua*
4. Conclusion

- Sum:
    - We can find that *Liangyi, Sixiang, Bagua* and hexagrams are lattice.
    - The way of interpretation *xiang*s, *gua*s and hexagrams is;
        - Take an element as n-tuple.
        - Substitute *yin* with 0, *yang* with 1.
        - Now you'll get the valuation of the element.

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
    1. *Liangyi*
    2. *Sixiang*
    3. *Bagua*
    4. *Hexagram*
3. Oppositions
    1. *Pang-tong gua*
    2. *Fan-dui gua*
4. Conclusion

- Now, let's consider the two oppositions.

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

- *pang-tong gua* is the result of exchanging all *yin* lines with *yang* and *yang* lines with *yin*.

yi
<1,0,0,0,1,1>

heng
<0,1,1,1,0,0>

- *Ex. yi* (益)'s *pang-tong gua* is *heng* (恒).

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

- Let the *pang-tong* operation be _.
- It is clear that the operation is as below;

a	1	0
a	0	1

- *Liangyi*'s *pang-tong* is simple; same as above.
- *Sixiang* can be represented as 2-tuple. *xiang*'s *pang-tong* <a,b> is equal to <a,b>.
  - Ex. *Shaoyin*'s *pang-tong* <1,0> is <1, 0>; that is <0,1>. This is *Shaoyang*.
- *Bagua* is 3-tuple. So *gua's pang-tong* <a,b,c> is equal to <a, b, c>.
  - Ex. *Zhen's pang-tong* <1, 0, 0> is <1, 0, 0>; that is <0,0,1>. This is *Xun*.
- Hexagrams can be expressed as 6-tuple. Its *pang-tong* <a,b,c,d,e,f> is <a, b, c, d, e, f>.

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

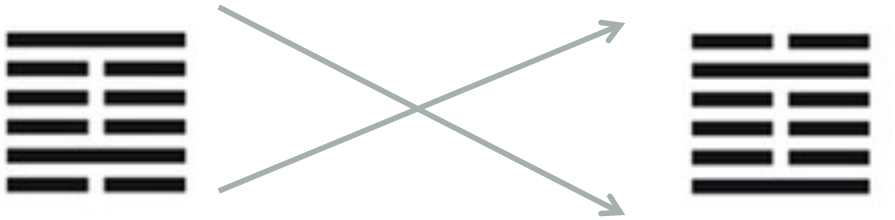
- We can generalize *pang-tong* operation;
- $\underline{< x_1, x_2, \ldots, x_n >} = <\underline{x_1}, \underline{x_2}, \ldots, \underline{x_n}>$.

a	1	0
$\underline{a}$	0	1

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

- *fan-dui gua* is the result of turning given symbol upside down.



meng
<0,1,0,0,0,1>

chun
<1,0,0,0,1,0>

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

- Let the *fan-dui* operation be *.
  - In *Liangyi*, *fan-dui* operation is identity operation; symbols don't change after the operation.
  - In *Sixiang*, $\langle x_1, x_2 \rangle^* = \langle x_2, x_1 \rangle$.
    - Ex. *Shaoyang*'s fan-dui is <0,1>*, that is <0,1>. This is *Shaoyin*.
  - In *Bagua*, $\langle x_1, x_2, x_3 \rangle^* = \langle x_3, x_2, x_1 \rangle$.
    - Ex. *Dui*'s fan-dui is <1,1,0>*, that is <0,1,1>. This is *Xun*.
  - In hexagrams,

    $\langle x_1, x_2, \ldots, x_6 \rangle^* = \langle x_6, x_5, \ldots, x_1 \rangle$.
    - Ex. *shike*'s fan-dui is <1,0,0,1,0,1>*, that is <1,0,1,0,0,1>. This is *jiaren*.

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
    1. *Liangyi*
    2. *Sixiang*
    3. *Bagua*
    4. *Hexagram*
3. Oppositions
    1. *Pang-tong gua*
    2. *Fan-dui gua*
4. Conclusion

- We can generalize *fan-dui* operation;
- $\langle x_1, x_2, \ldots, x_n \rangle^* = \langle x_n, x_{n-1}, \ldots, x_1 \rangle$.

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
    1. *Liangyi*
    2. *Sixiang*
    3. *Bagua*
    4. *Hexagram*
3. Oppositions
    1. *Pang-tong gua*
    2. *Fan-dui gua*
4. Conclusion

- We can see that *I Ching* can be interpreted as having mathematical structures.
- Is there other thoughts that have more excellent mathematical structures than *I Ching*?

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
   1. *Liangyi*
   2. *Sixiang*
   3. *Bagua*
   4. *Hexagram*
3. Oppositions
   1. *Pang-tong gua*
   2. *Fan-dui gua*
4. Conclusion

- Additional:
  - Leibniz thinks that these symbols can be interpreted by the binary system.
    - Ex. *Yin* (- -) is 0, and *yang* (—) is 1.
    - *Laoyin* <0,0> can be expressed 00, this is 0 in the system. *Shaoyang* <0,1> is 01, *Shaoyin* <1,0> is 10, *Laoyang* <1,1> is 11, so they are 1, 2 and 3.
    - In *Bagua* and hexagrams we can interpret each symbols in the same way.
  - By this method we can interpret all symbols as numbers, not as tuples. It seems more elegant.

3: From the mathematical point of view

1. What is lattice?
2. They are lattices
    1. *Liangyi*
    2. *Sixiang*
    3. *Bagua*
    4. *Hexagram*
3. Oppositions
    1. *Pang-tong gua*
    2. *Fan-dui gua*
4. Conclusion

- Additional;
    - But we don't take Leibniz's way.
    - The reason why we don't take this way is
        1. This may put you into confusion because you should add new notion to our method.
        2. In *I Ching*, hexagrams are not exhibited in the order of Leibniz; For example, there is no reason to represent a hexagram *xun* (䷸) as 63. This symbol appears second.

Conclusion

- Return to the first question; are eastern thoughts irrational?
- →No!
  - *I Ching* has been one of the most influential books in China. It has been a framework of thought for a long time.
  - This *I Ching* commits to mathematics and numbers.
    1. In *I Ching,* numbers symbolize (indirectly) everything in the world.
    2. We can interpret the structures in *I Ching* as lattices, important mathematical notion.
       - It shows that in China people have thought in a kind of mathematical (that is rational!) way, no matter whether they are voluntary or involuntary.
  - People have thought rationally in East!